Electrically charged pulsars

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In the present work we investigate one possible variation on the usual electrically neutral pulsars: the inclusion of electric charge. We study the effect of electric charge in pulsars assuming that the charge distribution is proportional to the energy density. All calculations were performed for zero temperature and fixed entropy equations of state.

PACS number(s):26.60.+c,24.10.Jv,21.65.+f,95.30.Tg

I. INTRODUCTION

Pulsars are believed to be the remnants of supernova explosions. They have masses 1–$2M_\odot$, radii $\sim 10\,$km, and a temperature of the order of 10^{11} K at birth, cooling within a few days to about 10^{10} K by emitting neutrinos. Pulsars are normally known as neutron stars. Qualitatively, a neutron star is analogous to a white dwarf star, with the pressure due to degenerate neutrons rather than degenerate electrons. The assumption that the neutrons in a neutron star can be treated as an ideal gas is not well justified: the effect of the strong force needs to be taken into account by replacing the equation of state (EoS) for an ideal gas by a more realistic EoS. The composition of pulsars remains a source of speculation, with some of the possibilities being the presence of hyperons [1, 2, 3], a mixed phase of hyperons and quarks [4, 5, 6, 7, 8], a phase of deconfined quarks or pion and kaon condensates [9]. Another possibility would be that pulsars are, in fact, quark stars [10]. In conventional models, hadrons are assumed to be the true ground state of the strong interaction. However, it has been argued [11, 12, 13, 14, 15] that *strange matter* composed of deconfined u, d and s quarks is the true ground state of all matter. In the stellar modeling, the structure of the star depends on the assumed EoS, which is different in each of the above mentioned cases. An important distinction between quark stars and conventional neutron stars is that the quark stars are self-bound by the strong interaction, whereas neutron stars are bound by gravity.

Once an adequate EoS is chosen, it is used as input to the Tolman-Oppenheimer-Volkoff (TOV) equations [16], which are derived from Einstein's equations in the Schwarzschild metric for a static, spherical star. Some of the stellar properties, as the radius, gravitational and baryonic masses, central energy densities, etc are obtained. These results are then tested against some of the constraints provided by astronomers and astrophysicists [17, 18] and some of the EoS are shown to be inappropriate for describing pulsars [5, 6, 9].

Notice also that the temperature in the interior of the star is not constant [8, 21], but the entropy per baryon is. This is the reason for choosing fixed entropies to take the temperature effects into account. The maximum entropy per baryon (S) reached in the core of a new born star is about 2 (in units of Boltzmann's constant) [19]. We then use EoS obtained with $S = 0$ ($T = 0$), 1 and 2.

We study the effects of the electric charge in compact stars. This study was first performed in stars composed of hot ionized gas [22] and then reconsidered for a cold star ($T = 0$) described by a polytropic EoS [20]. The electric charge distribution is assumed to be proportional to the mass density. The TOV equations again have to be modified to take the electric field into account. In the present work once more all possible classes of pulsars are examined in the presence of the electric field for $S = 0, 1$ and 2.

This paper is organized as follows: in Sec. II the formalisms of the electrically charged stars are revisited and the results are presented. In Sec. III the results are discussed and the main conclusions are drawn.

II. FORMALISM AND RESULTS

As the first step we need to know the EoS of the system,

$$\epsilon = \epsilon(p), \qquad n = n(p),$$

where p is the pressure, ϵ is the energy density, and n is the number density of baryons. Once an adequate EoS is obtained, it can be used to provide the stellar properties.

A. Electrically charged compact stars

In this section we include modifications in the TOV equations to describe electrically charged pulsars with null angular velocity. The geometry that describes a static spherical star is given by equation (1). In order that the Maxwell equations are incorporated into the stress tensor T^μ_ν, it becomes:

$$T^\mu_\nu = (p+\epsilon)u^\mu u_\nu - p\delta^\mu_\nu + \frac{1}{4\pi}\left(F^{\mu\alpha}F_{\alpha\nu} - \frac{1}{4}\delta^\mu_\nu F_{\alpha\beta}F^{\alpha\beta}\right), \tag{1}$$

where again p is the pressure, ϵ is the energy density, and u^μ is the 4-velocity vector.

The electromagnetic field obeys the relation

$$\left[\sqrt{-g}F^{\mu\nu}\right]_{,\nu} = 4\pi j^\mu \sqrt{-g}, \tag{2}$$

where j^μ is the four current density. Next we consider static stars only. Hence the electromagnetic field is only

due to the electric charge, which means that $F^{01} = -F^{10}$, and the other terms are absent. From the four-potential A_μ the surviving potential is $A_0 = \phi$. Thus the electric field is given by

$$E(r) = \frac{1}{r^2}\int_0^r 4\pi r^2 j^0 e^{(\nu+\lambda)/2} dr, \tag{3}$$

where $j^0 e^{\nu/2} = \rho_{ch}$ is the charge density. The electric field can be written as

$$\frac{dE(r)}{dr} = -\frac{2E}{r} + 4\pi\rho_{ch} e^{\lambda/2} \tag{4}$$

and total charge of the system as

$$Q = \int_0^R 4\pi r^2 \rho_{ch} e^{\lambda/2} dr, \tag{5}$$

where R is the radius of the star.

In the star frame the mass is

$$\frac{dM_{tot}}{dr}(r) = 4\pi r^2 \left(\epsilon + \frac{E(r)^2}{8\pi}\right). \tag{6}$$

To an observer at infinity, the mass is

$$M_\infty = \int_0^\infty 4\pi r^2 \left(\epsilon + \frac{E(r)^2}{8\pi}\right) dr = M_{tot}(R) + \frac{Q(R)^2}{2R} \tag{7}$$

By using the conservation law of the stress tensor ($T^\mu_{\nu;\mu} = 0$) we obtain the hydrostatic equation

$$\begin{aligned} \frac{dp}{dr} &= -\frac{\left[M_{tot} + 4\pi r^3 \left(p - \frac{E(r)^2}{8\pi}\right)\right](\epsilon + p)}{r^2\left(1 - \frac{2M_{tot}}{r}\right)} \\ &+ \rho_{ch} E(r) e^{\lambda/2}. \end{aligned} \tag{8}$$

The first term on the right-hand side comes from the gravitational force and the second term comes from the Coulomb force. By using the metric and the relation

$$R^\mu_\nu - \frac{1}{2} R \delta^\mu_\nu = -8\pi T^\mu_\nu, \tag{9}$$

we obtain the following differential equation

$$\frac{d\lambda}{dr} = \left[8\pi r e^\lambda \left(\epsilon + \frac{E(r)^2}{8\pi}\right) - \left(\frac{e^\lambda - 1}{r}\right)\right], \tag{10}$$

which is used to determine the metric e^λ.

So, we have a set of differential equations to be solved formed by equations(19), (21), (23) and (25). The boundary conditions at $r = 0$ are $E(r) = 0$, $e^\lambda = 1$, $n = \rho_c$ and at $r = R$, $p = 0$. We assume that the charge goes with the energy density ϵ as prescribed in [20]:

$$\rho_{ch} = f \times 0.86924 \times 10^3 \epsilon. \tag{11}$$

This choice of charge distribution is a reasonable assumption in the sense that a large mass can hold a large amount of charge. In table I results for electrically charged neutron stars are presented. We have calculated the results for 48 different configuration models of compact stars. Once again the EoS for hadronic and hybrid stars were taken from [8], the EoS for quarkionic stars were taken from [10]. In table I the electric charge Q is given in Coulomb and f varies from zero (no charge) to a small value (0.0006). The related mass-radius plots for hadronic, hybrid and quarkionic stars are given respectively in figs. 1,2 and 3-4.

III. RESULTS AND CONCLUSIONS

Let's now go back to our results in order to compare them with what is found in the literature and draw the conclusions.

In [23] the models named O,P,Q and R given in table I can again be compared with our result for the hadronic star at $T = 0$ and they are indeed very similar.

We next look at the results displayed in table I and corresponding figures. The general trend is the same observed in a simple polytropic EoS for $T = 0$ [20], i.e., the electric charge, the maximum mass and the mass observed at infinity increase with f, as it should be. Although the EoS used in the present work are very different from the one used in [20] the values of the radii obtained and the electric charge for a fixed f value are compatible. Figs. 1,2,3 and 4 also show the same behavior as fig. 2 of [20], i.e., as f increases, the maximum mass and radius of a family of stars increase. The EoS used in the present work are for bare pulsars, i.e. the outmost layer is not included. From table I one can see that the effect of entropy on a charged star remains the same as in a neutral star: the maximum masses and the radii decrease with the increase of the entropy for hadronic and quarkionic stars within the NJL model. For hybrid and MIT stars the behavior is not so well defined.

In the present work we have investigated one possible variation on the usual electrically neutral pulsars: the inclusion of electric charge. We have observed that the behaviors shown in previous works with much simpler EoS were also observed here. The influence of the temperature was also investigated. We are now in a position to calculate the energy released from the conversion of a metastable star (hadronic or hybrid) to a stable star (hybrid or quarkionic) under the influence of the electric charge. A more detailed and complete investigation, with a smaller bag parameter in the MIT bag model is under way.

Acknowledgments

This work was partially supported by CNPq (Brazil). M.D.A. would like to thank CNPq for a master's degree scholarship.

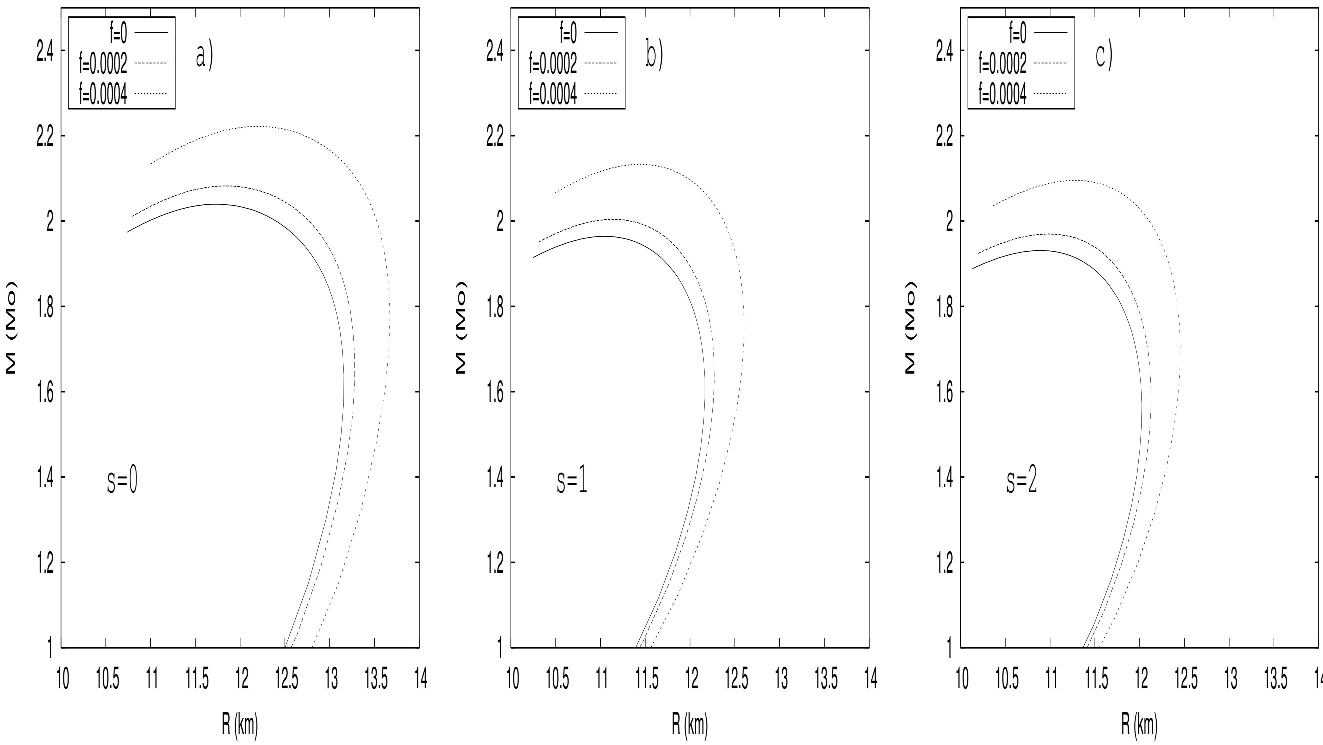

FIG. 1: Solutions for electrically charged hadronic stars with different values of f.

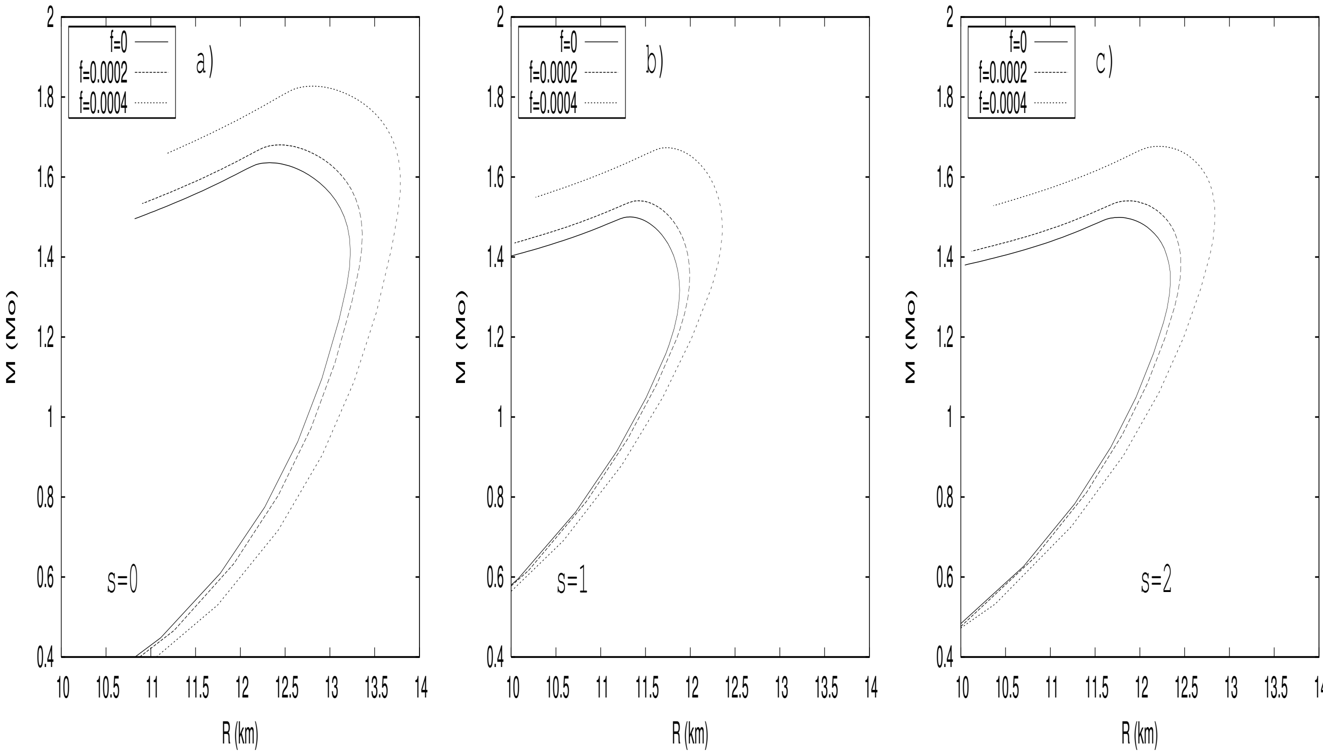

FIG. 2: Solutions for electrically charged hybrid stars with different values of f.

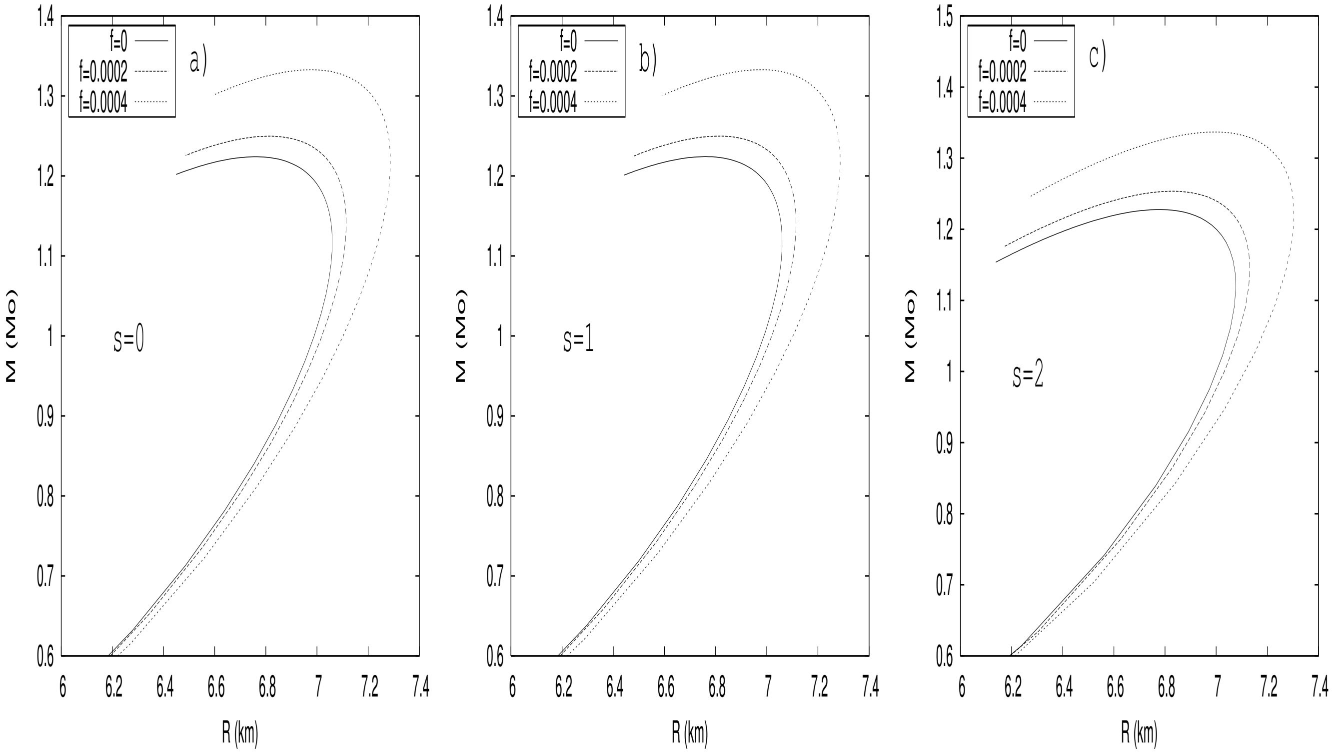

FIG. 3: Solutions for electrically charged quarkionic stars obtained with the MIT bag model for different values of f.

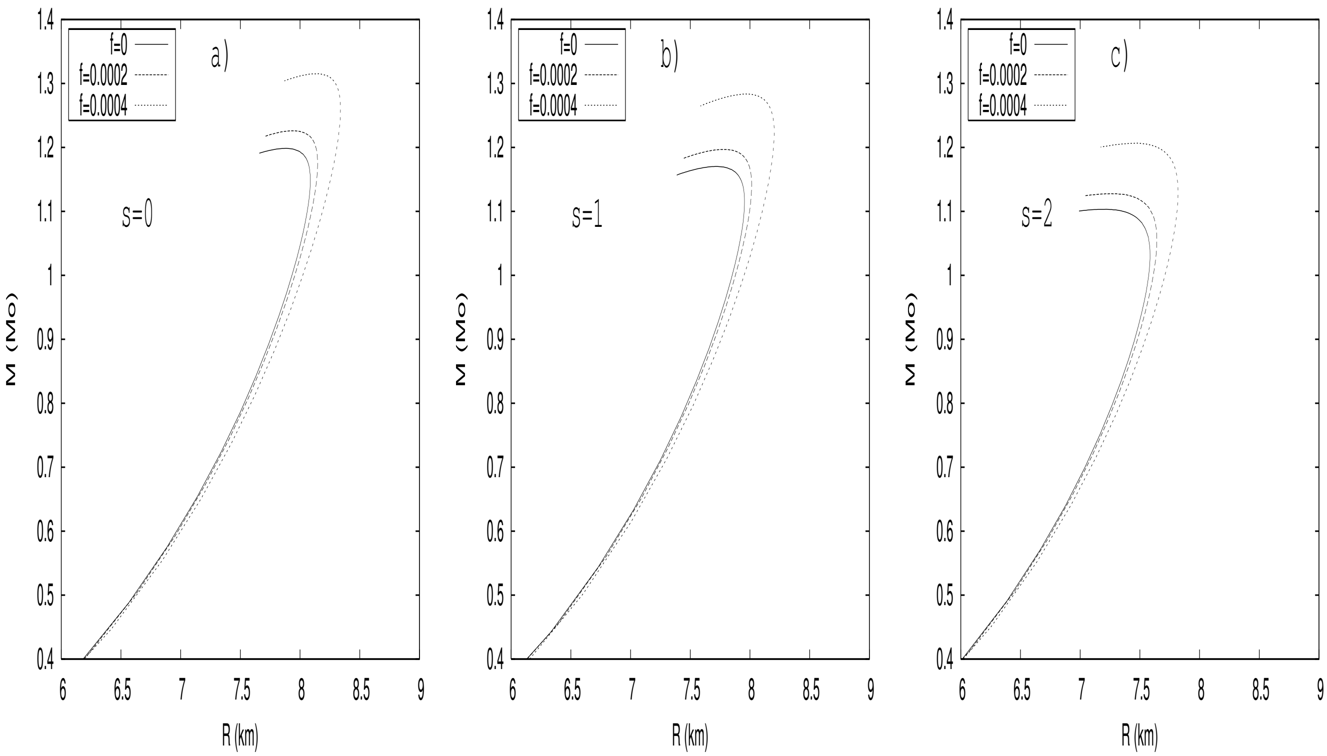

FIG. 4: Solutions for electrically charged quarkionic stars obtained with the NJL model for different values of f.

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TABLE I: Electrically compact stars with different charge fraction f.

Type	Entropy	f	$M_{\max}$ ($M_{\odot}$)	M_{∞} ($M_{\odot}$)	R (km)	ϵ_c (g/cm^3)	Q (C)
Hadronic	0	0	2.04	2.04	11.72	1.98×10^{15}	0
Hadronic	0	0.0002	2.08	2.10	11.84	1.94×10^{15}	7.97×10^{19}
Hadronic	0	0.0004	2.22	2.28	12.18	1.84×10^{15}	1.71×10^{20}
Hadronic	0	0.0006	2.50	2.66	12.73	1.75×10^{15}	2.91×10^{20}
Hadronic	1	0	1.96	1.96	11.02	2.23×10^{15}	0
Hadronic	1	0.0002	2.00	2.02	11.15	2.13×10^{15}	7.69×10^{19}
Hadronic	1	0.0004	2.13	2.19	11.44	2.04×10^{15}	1.64×10^{20}
Hadronic	1	0.0006	2.39	2.55	12.01	1.85×10^{15}	2.78×10^{20}
Hadronic	2	0	1.93	1.93	10.91	2.24×10^{15}	0
Hadronic	2	0.0002	1.97	1.98	11.01	2.19×10^{15}	7.55×10^{19}
Hadronic	2	0.0004	2.09	2.15	11.26	2.15×10^{15}	1.61×10^{20}
Hadronic	2	0.0006	2.34	2.50	11.86	1.90×10^{15}	2.72×10^{20}
hybrid	0	0	1.64	1.64	12.33	1.57×10^{15}	0
hybrid	0	0.0002	1.68	1.69	12.43	1.57×10^{15}	5.98×10^{19}
hybrid	0	0.0004	1.82	1.86	12.82	1.48×10^{15}	1.31×10^{20}
hybrid	0	0.0006	2.13	2.23	13.52	1.39×10^{15}	2.31×10^{20}
hybrid	1	0	1.50	1.50	11.32	1.75×10^{15}	0
hybrid	1	0.0002	1.54	1.55	11.43	1.71×10^{15}	5.44×10^{19}
hybrid	1	0.0004	1.67	1.70	11.74	1.66×10^{15}	1.19×10^{20}
hybrid	1	0.0006	1.94	2.03	12.34	1.58×10^{14}	2.10×10^{20}
hybrid	2	0	1.50	1.50	11.76	1.58×10^{15}	0
hybrid	2	0.0002	1.54	1.55	11.86	1.58×10^{15}	5.41×10^{19}
hybrid	2	0.0004	1.68	1.71	12.21	1.53×10^{15}	1.18×10^{20}
hybrid	2	0.0006	1.95	2.04	12.86	1.44×10^{14}	2.10×10^{20}
Quarkonic(MIT)	0	0	1.22	1.22	6.77	5.14×10^{15}	0
Quarkonic(MIT)	0	0.0002	1.25	1.26	6.81	5.13×10^{15}	4.75×10^{19}
Quarkonic(MIT)	0	0.0004	1.33	1.37	6.97	4.95×10^{15}	1.02×10^{20}
Quarkonic(MIT)	0	0.0006	1.50	1.60	7.28	4.56×10^{15}	1.74×10^{20}
Quarkonic(MIT)	1	0	1.22	1.22	6.76	5.17×10^{15}	0
Quarkonic(MIT)	1	0.0002	1.25	1.26	6.82	5.07×10^{15}	4.75×10^{19}
Quarkonic(MIT)	1	0.0004	1.33	1.37	6.98	4.88×10^{15}	1.02×10^{20}
Quarkonic(MIT)	1	0.0006	1.50	1.60	7.29	4.50×10^{15}	1.74×10^{20}
Quarkonic(MIT)	2	0	1.23	1.23	6.79	5.08×10^{15}	0
Quarkonic(MIT)	2	0.0002	1.25	1.26	6.83	5.09×10^{15}	4.77×10^{19}
Quarkonic(MIT)	2	0.0004	1.34	1.37	6.98	4.94×10^{15}	1.03×10^{20}
Quarkonic(MIT)	2	0.0006	1.50	1.60	7.31	4.49×10^{15}	1.74×10^{20}
Quarkonic(NJL)	0	0	1.20	1.20	7.87	3.45×10^{15}	0
Quarkonic(NJL)	0	0.0002	1.23	1.23	7.93	3.45×10^{15}	4.42×10^{19}
Quarkonic(NJL)	0	0.0004	1.32	1.34	8.14	3.30×10^{15}	9.54×10^{19}
Quarkonic(NJL)	0	0.0006	1.49	1.57	8.48	3.20×10^{15}	1.64×10^{20}
Quarkonic(NJL)	1	0	1.17	1.17	7.71	3.68×10^{15}	0
Quarkonic(NJL)	1	0.0002	1.19	1.20	7.79	3.56×10^{15}	4.32×10^{19}
Quarkonic(NJL)	1	0.0004	1.28	1.31	8.00	3.50×10^{15}	9.31×10^{19}
Quarkonic(NJL)	1	0.0006	1.45	1.53	8.31	3.38×10^{15}	1.60×10^{20}
Quarkonic(NJL)	2	0	1.10	1.10	7.18	4.58×10^{15}	0
Quarkonic(NJL)	2	0.0002	1.13	1.13	7.29	4.34×10^{15}	4.09×10^{19}
Quarkonic(NJL)	2	0.0004	1.20	1.23	7.42	4.40×10^{15}	8.23×10^{19}
Quarkonic(NJL)	2	0.0006	1.36	1.43	7.80	3.98×10^{15}	1.50×10^{20}